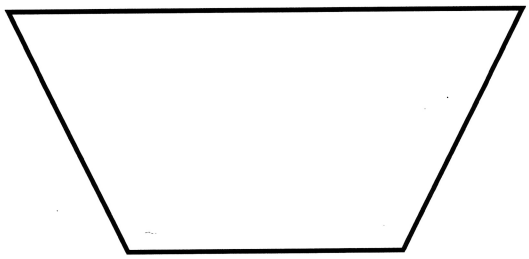
Homework: pg 452 #1, 2, 3, 4ace, 5, 6, 10, 13, (15)

Arithmetic Series

Series – the sum of the elements ~~of~~ in a sequence.

Problem

The new Lourdes auditorium will have a section of seats arranged like the following:



The number of seats in each subsequent row starting from the front is listed as an arithmetic sequence: 20, 23, 26, 29,…., 68, 71.

a) How many rows are there in this section?

71 is the 18th term

$t_n = a + (n-1)d$

$= 20 + (n-1)(3)$

$= 20 + 3n - 3$

$t_n = 17 + 3n$

$t_n = 71$

$71 = 17 + 3n$

$71 - 17 = 3n$

$\frac{54}{3} = \frac{3n}{3}$

$n = 18$

∴ There are 18 rows.

b) How many seats are there total in the section?

# of seats $= 20 + 23 + 26 + 29 + \ldots + 71$

We typically use the letter "S" to denote a series.

Write the series backwards

$S = 20 + 23 + 26 + 29 + \ldots + 71$

$S = 71 + 68 + 65 + 62 + \ldots + 20$

$2S = 91 + 91 + 91 + 91 + \ldots + 91$

$\frac{2S}{2} = \frac{18(91)}{2}$

$S = 819$ seats

The Sum of the First 'n' Terms of an Arithmetic Series

$$S_n = \frac{n(t_1+t_n)}{2}$$

- S_n is the sum of the first n terms.
- n is the number of terms.
- t_1 is the first term.
- t_n is the nth term.

Example 1

Find the sum of the first 37 terms of an arithmetic series whose first term is 12 and the common difference is 9.

$t_n = a + (n-1)d$

$= 12 + (n-1)9$

$= 12 + 9n - 9$

$t_n = 3 + 9n$

$t_{37} = 3 + 9(37)$

$t_{37} = 336$

$S_n = \frac{n(t_1 + t_n)}{2}$

$S_{37} = \frac{37(12 + t_{37})}{2}$

$= \frac{37(12 + 336)}{2}$

$= 6438$

Use the previous equation for an arithmetic series to derive a new formula that incorporates the common difference 'd' instead of the value of the n^{th} term.

$S_n = \frac{n(t_1 + t_n)}{2}$ $\quad t_n = a + (n-1)d$

$= \frac{n[a + a + (n-1)d]}{2}$

$$S_n = \frac{n[2a + (n-1)d]}{2}$$

Example 2

Consider the series 5, 9, 13, 17, …

What is the sum of the first 20 terms?

$a = 5$

$n = 20$

$d = 9 - 5$

$= 4$

$S_n = \frac{n[2a + (n-1)d]}{2}$

$S_{20} = \frac{20[2(5) + (20-1)(4)]}{2}$

$= \frac{20[10 + 76]}{2}$

$= 860$

Which of one of these statements is correct?

$S_{10} - S_9 = S_8$ ×

$S_{10} - S_9 = t_{10}$ ✓

$S_{10} - S_9 = S_1$ ×

$S_{10} - S_9$

$= (t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 + t_9 + t_{10})$
$\quad - (t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 + t_9)$

$= t_{10}$