**Homework: pg 424 #1, 2, 3, 4, 5i, ii, 6, 7ii, 8i, iii, 10, 13, 14, 15**

## Arithmetic Sequences

**Sequence -** an ordered list of numbers.

1. Determine the next three numbers in each sequence:

   a) 5, 7, 9, 11, 13, 15, 17, 19, 21
   b) 1.5, 3, 6, 12, 24, 48, 96, 192, 384
   c) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 ← Fibonacci Sequence
   d) 2, 3, 5, 7, 11, 13, 17, 19, 23
   e) 1, 4, 9, 16, 25, 36, 49, 64, 81
   f) $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$
   g) 3, 1, 4, 1, 5, 9, 2, 6, 5, 4 ← Pi ($\pi$)

**Arithmetic Sequence** - is a sequence such that each term is obtained by adding a constant to the previous term.

Ex; 4, 7, 10, 13, 16, 19, ... or 57, 55, 53, 51, 49, 47, ...

**Geometric Sequence** - is a sequence such that each term is obtained by multiplying a constant factor by the previous term.

Ex; 2, 6, 18, 54, 162, ... or 64, 32, 16, 8, 4, ...

2. From the above, ...

   a) Which sequences are arithmetic? a)

   b) Which sequences are geometric? b)

   c) Which sequences are neither?

   $\underbrace{\text{c), d), e), f), g)}}_{\text{all others}}$

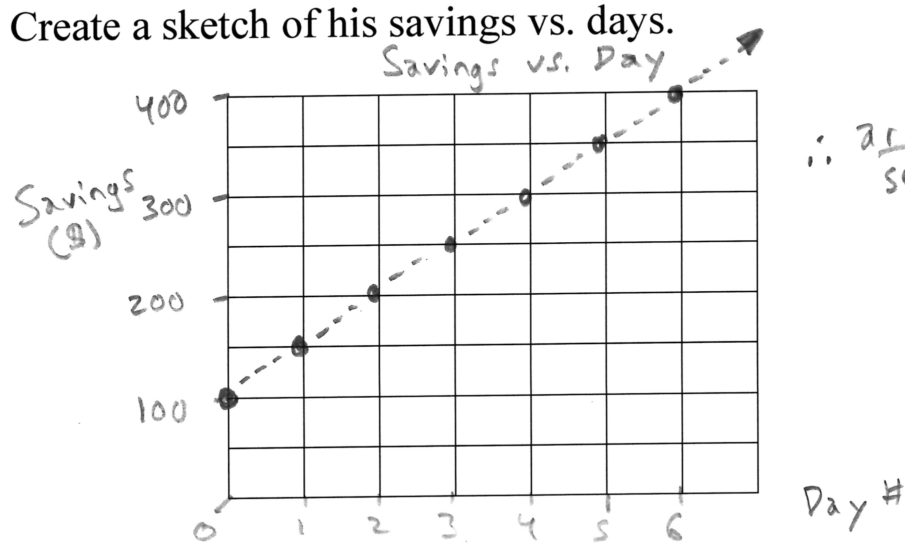
**Example 1**

Big Bird has a part-time job at the Quik-E-Mart. He earns $50 per day. He starts with $100 in his savings account. Each day, he adds the $50 he earned to his total savings.

a) How much will Big Bird have saved after 6 days? Complete the table below.

| Day | Savings |
|---|---|
| 0 | 100 |
| 1 | 150 |
| 2 | 200 |
| 3 | 250 |
| 4 | 300 |
| 5 | 350 |
| 6 | 400 |

b) Create a sketch of his savings vs. days.




∴ arithmetic sequences grow linearly

**Discrete Function** - uses numbers with spaced intervals between that are not defined; represented by dots on a graph.
Ex; a golf score, pages in a book, shoe sizes, etc...

**Continous Function** - uses all numbers including the space between; represented by a continuous line/curve on a graph.
Ex; your height, volume of rain collected, etc…

## Arithmetic Sequences and the Common Difference

In an arithmetic sequence, the number obtained by subtracting any term from the next is a constant; this constant is called the common difference, d, and it represents the linear change from one term to the next.

### Example 2

For each arithmetic sequence, determine the common difference 'd'.

a) 100, 130, 160, 190,... $d = t_3 - t_2 = 160 - 130 = 30$

b) 12, 2, -8, -18, ... $d = t_4 - t_3 = (-18) - (-8) = -10$

c) 17, 17.5, 18, 18.5,... $d = 0.5$

### Example 3

For the arithmetic sequence: 2, 9, 16, ...

a) Determine the 20th term.

$2 \overset{+d}{,} 9 \overset{+d}{,} 16 \overset{+d}{,} \ , \ ,$

$t_{20} = 2 + 19d$

$t_{20} = 2 + 19(7)$

$= 2 + 133$

$= 165$

b) Suppose you know the term number, how can you calculate the term?

any term = First term + (term # − 1)d

### Example 4

If the 5th term in an arithmetic sequence is 19 and the 8th term is 61, what are the 6th and 7th term?

$19 + 3d = 61$

$3d = 61 - 19$

$\frac{3d}{3} = \frac{42}{3}$

$d = 14$

_ _ _ _ $\underset{5^{th}}{19} \overset{+d}{\ } 33 \overset{+d}{\ } 47 \overset{+d}{\ } \underset{8^{th}}{61}$ _

**Example 5**

Consider the following arithmetic sequence:

$$2, 5, 8, 11, 14, 17, 20, \ldots$$

The first term, a, in this sequence is 2. The common difference is 3.

a = 2 ← first term

d = 3

The given arithmetic sequence can instead be expressed as:

2, 2+3, 2+3+3, 2+3+3+3, 2+3+3+3+3, etc… which is the same as:

2+0(3), 2+1(3), 2+2(3), 2+3(3), 2+4(3), 2+5(3), 2+6(3), …

If we use the variables 'a' and 'd', we get…

a+0d, a+1d, a+2d, a+3d, a+4d, a+5d, a+6d, …

From observations of this sequence, we can create a general formula for the nth term of an arithmetic sequence as follows.

**The general term of an Arithmetic Sequence**

$\boxed{t_n = a + (n-1)d}$

**where**

- **a is the first term**
- **n is the term number**
- **d is the common difference**

## Example 6

Given the following terms of an arithmetic sequence (4, 19, 34, …), determine the 82nd term.

$t_n = a + (n-1)d$
$t_n = 4 + (n-1)15$
$t_n = 4 + 15n - 15$
$t_n = -11 + 15n$

$t_{82} = -11 + 15(82)$
$= -11 + 1230$
$= 1219$

## Example 7

If the first term of an arithmetic sequence is 8 and the tenth term is 71, what is the 14th term?

$\boxed{a = 8 \quad d = ?}$

$t_n = a + (n-1)d$
$t_{10} = 8 + (10-1)d$
$71 = 8 + 9d$
$71 - 8 = 9d$

$\frac{63}{9} = \frac{9d}{9}$
$d = 7$

$t_n = 8 + (n-1)(7)$
$t_n = 8 + 7n - 7$
$t_n = 1 + 7n$
set $n = 14$
$t_{14} = 1 + 7(14)$
$t_{14} = 99$

## Recursive formulas

It is also possible to express terms in an arithmetic sequence as a recursive formula; each term is expressed in relation to previous term(s). If the notation $t_n$ refers to some term in a sequence then $t_{n+1}$ would be the term after $t_n$. The term $t_{n-1}$ would be the term that appears before $t_n$ in the sequence.

For example, if n = 8 then
$t_n = t_8$
$t_{n+1} = t_{8+1} = t_9$ ← next term
$t_{n-1} = t_{8-1} = t_7$ ← previous term

## Example 8

Create a recursive formula to define the following arithmetic sequences:

a) 3, 12, 21, 30, 39,.....

$t_1 = 3, \quad t_n = t_{n-1} + 9, \quad n > 1$

b) 402, 398, 394, 390, 386,....

$t_1 = 402, \quad t_n = t_{n-1} - 4, \quad n > 1$

Part 1 Part 2 Part 3