

Applications of Sinusoidal Functions

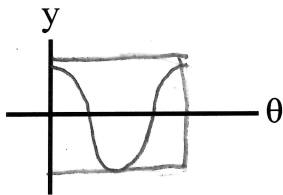
The equation of a sinusoidal function can be determined from its graph:

1. Draw a box around one cycle of a sinusoidal.
2. The value for 'k' can be determined from the period (one cycle) of the sinusoidal of measured from peak to peak. Then use the equation $k = \frac{360^\circ}{T}$
3. The value 'c' is the location of the line of equilibrium or use $c = \frac{\text{max} + \text{min}}{2}$
4. The phase shift 'd' is the horizontal distance to the left of the box.
5. The value of 'a', the vertical expansion/compression, is closely related to the amplitude since the amplitude = $|a|$; use the amplitude for a then assign a negative if the graph is a reflected sine or cosine curve about the independent axis.

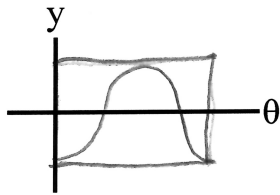
Activity

Create a simple basic sketch of the following sinusoidal functions:

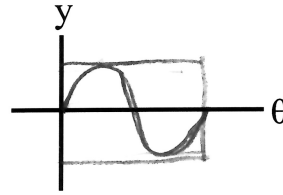
$$y = \cos \theta$$



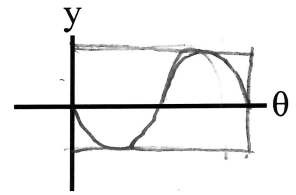
$$y = -\cos \theta$$



$$y = \sin \theta$$



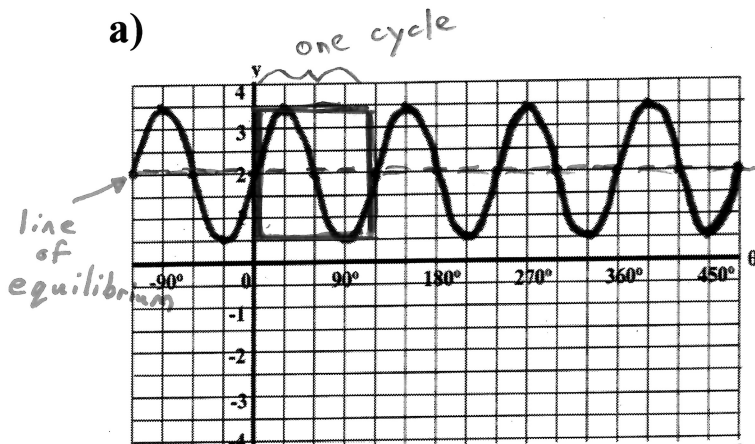
$$y = -\sin \theta$$



Example 1

Determine an equation for each sinusoidal graph below:

a)



$$k = \frac{360^\circ}{T} = \frac{360^\circ}{120^\circ}$$

$$k = 3$$

$$d = 0$$

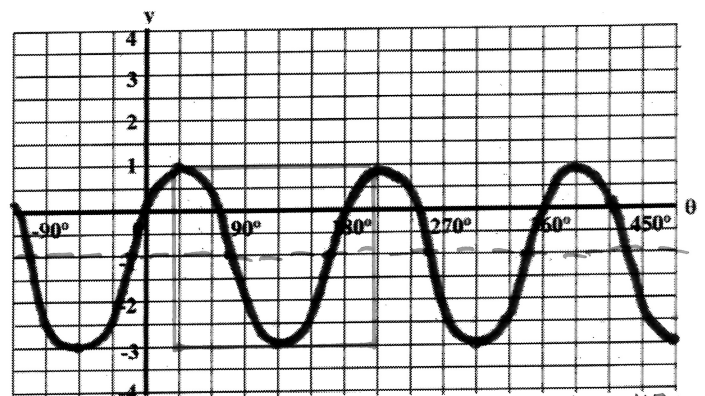
$$a = 1.5$$

$$c = 2$$

$$y = a \sin[k(\theta - d)] + c$$

$$y = 1.5 \sin(3\theta) + 2$$

b)



$$k = \frac{360^\circ}{T} = \frac{360^\circ}{180^\circ}$$

$$k = 2$$

$$d = 30^\circ$$

$$a = 2$$

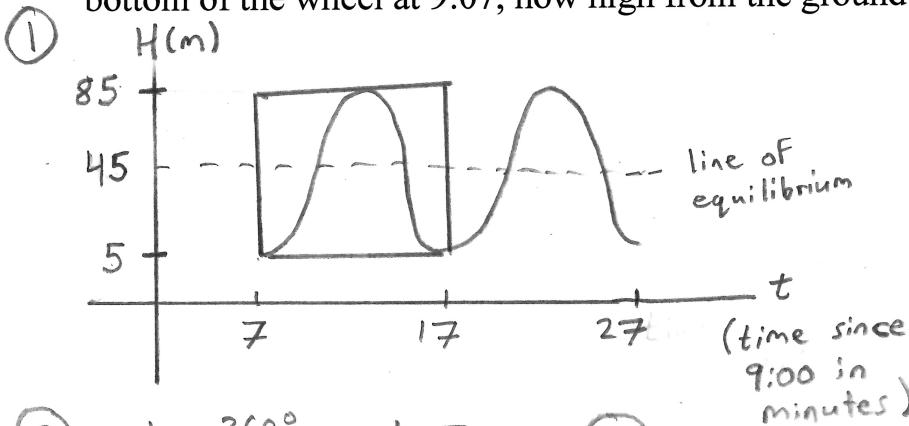
$$c = -1$$

$$y = a \cos[k(\theta - d)] + c$$

$$y = 2 \cos[2(\theta - 30^\circ)] - 1$$

Example 2

The first Ferris wheel built in 1893 had a diameter of 80 m. The base of the wheel was 5 m above the ground. It took 20 minutes to do 2 full revolutions. If Milton boarded the bottom of the wheel at 9:07, how high from the ground would he be at 9:20?



②

$$K = \frac{360^\circ}{T}$$

$$= \frac{360^\circ}{10}$$

$$K = 36$$

$$d = 7$$

$$a = -40$$

$$c = 45$$

③

$$H = a \cos [K(t-d)] + c$$

$$H = -40 \cos [36(t-7)] + 45$$

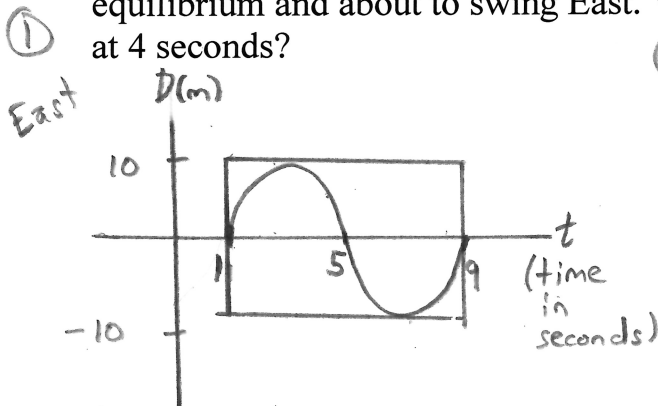
④ set $t = 20$

$$H = -40 \cos [36(20-7)] + 45$$

$$H \approx 57.4 \text{ m}$$

Example 3

The top of a building designed by Fractals Incorporated sways East (10 m) then West (-10 m) of its stable equilibrium position. One full swing from equilibrium to the East, to the West then back to equilibrium takes 8 seconds. At $t = 1$ s, the building is at equilibrium and about to swing East. What is the displacement at the top of the building at 4 seconds?



③

$$D = a \sin [K(t-d)] + c$$

$$D = 10 \sin [45(t-1)] + 0$$

set $t = 4$

④

$$D = 10 \sin [45(4-1)]$$

$$= 10 \sin [135^\circ]$$

$$= 7.1 \text{ m [East]}$$

* careful
If $D = -8 \text{ m}$, then
You can't write
 -8 m [West] as that
would imply East.
Instead, you should
write 8 m [West]

②

$$K = \frac{360^\circ}{T}$$

$$= \frac{360^\circ}{8}$$

$$= 45$$

$$d = 1$$

$$a = 10$$

$$c = 0$$