Hmwk: pg 261 #1bc, 6, 9, 10c, 11, pg 269 # 14

Applications of Exponential Functions: Part 2

Doubling Time

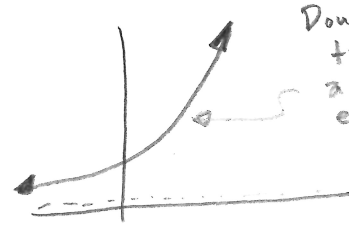
Doubling time refers to a type of exponential growth whereby some quantity doubles for each constant interval of time. If the initial amount of some quantity is known to double every 'd' units of time, then the future amount can be represented by the function:

$$y = a(2)^{\frac{t}{d}}$$

times expressed with the same units

where

- **'a'** is the initial amount
- **'t'** is the time elapsed
- **'d'** is the doubling time
- **'y'** is the future amount



Doubling time is a form of exponential growth.

Example 1

A post secondary graduate puts $2500 into a high interest savings account. The value of the investment is expected to double every 8 years. How much will the investment be worth in 30 years?

$y = a(2)^{t/d}$

$y = 2500(2)^{t/8}$

set $t = 30$

$y = 2500(2)^{30/8}$

$\boxed{\$ = 33635.86}$

Example 2

The number of bacteria in a petrie dish can double every 10 hours. If there is initially 1000 bacteria cells, how many will there be in 2 days?

$y = a(2)^{t/d}$

$y = 1000(2)^{t/10}$

set $t = 48$

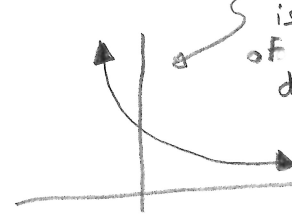
$y = 1000(2)^{48/10}$

$y \approx 27858$ bacteria

Half-life

Half-life refers to a type of exponential decay whereby some quantity is reduced by a factor of one half at every constant interval of time. If the initial amount of some quantity is known to drop by a factor of one half every 'h' units of time, then the future amount can be represented by the function:

$$y = a(0.5)^{\frac{t}{h}}$$



Half-life is a form of exponential decay.

where

- **'a'** is the initial amount
- **'t'** is the time elapsed
- **'h'** is the half-life
- **'y'** is the future amount

Example 1

On a cool spring day in Guelph, the outside temperature is 0°C.
Mr. Ryan puts his cup of tea on the top of his car then goes inside. Oops, he forgot his tea. The temperature of the tea is initially 80°C. If the half-life of the tea's temperature is 12 minutes, what will be the temperature of the tea after half an hour?

$y = a(0.5)^{t/d}$

$y = 80(0.5)^{t/12}$

set $t = 30$ minutes

$y = 80(0.5)^{(30/12)}$

$y \cong 14.1°C$

Example 2

The half-life of carbon-14 in a ~~recently~~ deceased organism is about 5700 years. If a corpse initially had 2000 C-14 atoms but now has 600 C-14 atoms, how long has the corpse been deceased?

$y = a(0.5)^{t/d}$

$\frac{600}{2000} = \frac{\cancel{2000}(0.5)^{t/5700}}{\cancel{2000}}$

$0.3 = (0.5)^{t/5700}$

$\log 0.3 = \frac{t}{5700} \log 0.5$

$\frac{\log 0.3}{\log 0.5} = \frac{\frac{t}{5700} \cancel{\log 0.5}}{\cancel{\log 0.5}}$

$\frac{t \cancel{\log 0.5}}{\cancel{\log 0.5}} = \frac{5700 \log 0.3}{\log 0.5}$

$t \cong 9901$ years