Homework: pg 261 # 1ad, 2ab, 3, 4, 5, 7, 8, 12, 13, 15

Applications of Exponential Functions: Part 1
Exponential Growth/ Decay

Exponential growth/decay is used to model several real-life scenarios:

- The increasing value of an investment.
- The depreciation of a car's value.
- The increase in population.
- The decrease in light intensity as it travels through water.
- The exponential growth of infected cases during a pandemic.

In general, exponential growth/decay relationships can be modeled by an equation of the form:

Exponential growth → $y = a(1 + r)^x$

Exponential decay → $y = a(1 - r)^x$

where

- a is the initial amount
- $\pm r$ is the percentage growth/decay rate respectively
- x is the number of periods of growth/decay
- y is the future amount

It is also possible to combine both equations above by writing them as a single equation as follows:

$$y = ab^x$$

where b (the growth/decay factor) is given by

- $b = 1 + r$ for exponential growth
- $b = 1 - r$ for exponential decay

Example 1

Initially, ten people are infected by a novel virus. The number of people infected grows exponentially by 9% each day. How many people will be infected after 100 days?

$$y = a(1+r)^t$$
$$y = 10(1+0.09)^t$$
$$y = 10(1.09)^t$$
set $t = 100$
$$y = 10(1.09)^{100}$$
$$y = 55290 \text{ people}$$

Example 2

Finnegan purchases a Saab for $48,000. The value of the car depreciates by 18% each year. How much will the car be worth after 5 years?

$$y = a(1-r)^t$$
$$y = 48000(1-0.18)^t$$
$$y = 48000(0.82)^t$$
set $t = 5$
$$y = 48000(0.82)^5$$
$$y = \$17795.51$$

Example 3

The intensity of light projected downwards into water drops exponentially by 8% for every meter in depth measured from the surface. If a waterproofed 60-Watt bulb is projected into a lake, how intense is the light at a depth of 7 m?

$$y = a(1-r)^D$$
$$y = 60(1-0.08)^D$$
$$y = 60(0.92)^D$$
set $D = 7$
$$y = 60(0.92)^7$$
$$y \cong 33 \text{ Watts}$$

Example 4

The population of Guelph is currently 135,000. The number of residents in Guelph is expected to increase by about 2.2% each year.

a) What is the projected population of Guelph in 15 years?

$$y = a(1+r)^t$$
$$y = 135000(1+0.022)^t$$
$$y = 135000(1.022)^t$$
set $t = 15$
$$y = 135000(1.022)^{15}$$
$$y = 187110 \text{ people}$$

b) When will the population of Guelph be 250,000?

$$y = 135000(1.022)^t$$
$$\frac{250000}{135000} = \frac{\cancel{135000}(1.022)^t}{\cancel{135000}}$$
$$\frac{250}{135} = (1.022)^t$$
$$\log\left(\frac{250}{135}\right) = \log(1.022^t)$$
$$\frac{\log\left(\frac{250}{135}\right)}{\log(1.022)} = \frac{t\cancel{\log(1.022)}}{\cancel{\log(1.022)}}$$
$$t \cong 28.3 \text{ years}$$