

Adding and Subtracting Rational Expressions

Recall:

Evaluate the following:

$$\begin{aligned}
 \text{a) } & \frac{2}{5} + \frac{4}{15} = \frac{2}{3} \\
 & = \frac{6}{15} + \frac{4}{15} \\
 & = \frac{10}{15}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{1}{3} - \frac{2}{7} \\
 & = \frac{7}{21} - \frac{6}{21} \\
 & = \frac{1}{21}
 \end{aligned}$$

Alternatively, the expressions can be evaluated using a cross method (not cross multiplication) for addition/subtraction as follows:

$$\begin{aligned}
 \text{a) } & \frac{2}{5} + \frac{4}{15} = \frac{2}{3} \\
 & = \frac{30 + 20}{75} \\
 & = \frac{50}{75}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{1}{3} - \frac{2}{7} \\
 & = \frac{7 - 6}{21} \\
 & = \frac{1}{21}
 \end{aligned}$$

Examples

Expand and simplify each rational expression below then state all restrictions:

$$\begin{aligned}
 \text{a) } & \frac{5x}{2x-1} + \frac{x-3}{2x-1} = 3 \\
 & = \frac{5x + (x-3)}{2x-1} \\
 & = \frac{6x-3}{2x-1} \\
 & = \frac{3(2x-1)}{2x-1}
 \end{aligned}$$

$2x-1 \neq 0$
 $\frac{2x-1}{2} \neq \frac{1}{2}$
 $x \neq \frac{1}{2}$

$$\begin{aligned}
 \text{b) } & \frac{3}{2x} - \frac{5}{5x^2} = \frac{3x-2}{2x^2} \\
 & = \frac{15x}{10x^2} - \frac{10}{10x^2} \\
 & = \frac{15x-10}{10x^2} \\
 & = \frac{5(3x-2)}{5 \cdot 2x^2}
 \end{aligned}$$

$x \neq 0$

$$\begin{aligned}
 \text{c) } & \frac{(5x-y)}{3x-y} - \frac{(4x+y)}{3x-y} \\
 & = \frac{(5x-y) - (4x+y)}{3x-y} \quad 3x-y \neq 0 \\
 & = \frac{5x-y-4x-y}{3x-y} \quad \frac{3x \neq y}{3 \quad 3} \\
 & = \frac{x-2y}{3x-y} \quad x \neq \frac{y}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } & \frac{3m+4}{mn} - \frac{1}{m} - \frac{2}{n} \\
 & = \frac{3m+4}{mn} - \frac{n}{mn} - \frac{2m}{mn} \\
 & = \frac{3m+4-n-2m}{mn} \\
 & \quad m \neq 0 \\
 & \quad n \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{g) } & \frac{3x+2}{x^2-3x-10} - \frac{x+8}{x^2-x-20} \\
 & = \frac{(x+4)(3x+2)}{(x+4)(x-5)(x+2)} - \frac{(x+8)(x+2)}{(x-5)(x+4)(x+2)} \\
 & = \frac{(3x^2+2x+12x+8) - (x^2+2x+8x+16)}{(x+4)(x-5)(x+2)} \\
 & = \frac{3x^2+2x+12x+8-x^2-2x-8x-16}{(x+4)(x-5)(x+2)} \\
 & = \frac{2x^2+4x-8}{(x+4)(x-5)(x+2)} \\
 & \quad x \neq -4, 5, -2
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{(2)(y-5)}{6} + \frac{(2y-3)(3)}{4(3)} \\
 & = \frac{2(y-5)}{12} + \frac{3(2y-3)}{12} \\
 & = \frac{2y-10+6y-9}{12} \\
 & = \frac{8y-19}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } & \frac{3x+9}{x^2-9} + \frac{5x}{x-3} \\
 & = \frac{3(x+3)}{(x-3)(x+3)} + \frac{5x}{x-3} \\
 & = \frac{3+5x}{x-3} \\
 & \quad x \neq \pm 3
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } & \frac{3x}{x+3} + \left(\frac{1}{x+2} \div \frac{x+3}{x^2-4} \right) \\
 & = \frac{3x}{x+3} + \left(\frac{1}{x+2} \cdot \frac{(x-2)(x+2)}{x+3} \right) \\
 & = \frac{3x}{x+3} + \frac{x-2}{x+3} \\
 & = \frac{4x-2}{x+3} \\
 & \quad x \neq -3, \pm 2
 \end{aligned}$$